

Università di Trieste
Corso di Studi in Matematica
Laurea Magistrale

Algebra Computazionale

docente: prof. Alessandro Logar

Ideals, modules, term order — Noetherian rings and modules. Hilbert basis theorem. Terms on $P = K[x_1, \dots, x_n]$ and on P^r . Term orders. Macaulay Basis Theorem. Algorithm of division of polynomials.

Graded rings — Graded rings, graded modules. Homomorphisms of graded rings. Shifts.

Gröbner basis, I — Definition of a σ - Gröbner basis. Characterizations of Gröbner bases.

Syzygies — Definition of syzygies. Syzygies of monomial modules and their computation.

Gröbner bases, II — Lifting of syzygies. Normal form of an element w.r.t. a Gröbner basis. Buchberger algorithm. Schreyer theorem for computing syzygies. Syzygies Hilbert theorem.

Properties of graded modules — Minimal sistem of generators of a graded module. Nakayama lemma (for homogeneous modules). Homogeneous Gröbner bases. FFR of a graded module over P . Betti numbers.

Hilbert function and Hilbert series for a graded module — Definition and properties of the Hilbert function. Computation of the HF via Macaulay basis theorem. Definition and properties of the Hilbert series.

Invariant polynomials — Symmetrical polynomials. Action of $GL(n, \mathbb{C})$ on P . Action of finite subgroups Γ of $GL(n, \mathbb{C})$ on P . Reynolds operator. Hilbert finiteness theorem. Molien theorem for computing the Hilbert series of P^Γ . Computation of the fundamental invariants of P^Γ .

Bibliography

1. Martin Kreuzer, Lorenzo Robbiano. *Computational Commutative algebra 1* Springer-Verlag.
2. Martin Kreuzer, Lorenzo Robbiano. *Computational Commutative algebra 2*, Springer-Verlag.
3. David Eisenbud. *Commutative Algebra vith a view toward Algebraic Geometry* Springer-Verlag.
4. Bernd Sturmfels. *Computational invariant theory*, Springer-Verlag.
5. M.F. Atiyah, I.G. Macdonald, *Commutative algebra*.