Numerical methods for Ordinary Differential Equations

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Timetable: 12 hrs. (Part I, Prof. Zennaro) First lecture on January 23, 2014, 15:00 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.
12 hours (Part II, Prof. Vermiglio), Lectures on February-March 2014 (to be defined).

Course requirements: it is advisable to have attended a basic course in Numerical Analysis.

Examination and grading: A unique written exam for both Part I and Part II.

SSD: MAT/08 Numerical Analysis

Aim: We present basic numerical methods for initial value problems in ordinary differential equations and we analyse their convergence and stability properties.

Course contents:

Part I

Existence and uniqueness of the solution and continuous dependence on the data for the initial value problem $y'(x) = f(x, y(x)), y(x_0) = y_0$.

Classical Lipschitz constant and right hand side Lipschitz constant.

General one-step methods; explicit and implicit Runge-Kutta methods.

Definition of local truncation and discretization error for one-step methods and definition of consistency of order p.

Convergence theorem with order p for one-step methods. Order conditions for Runge-Kutta methods. Order barriers for explicit and implicit methods.

Variable stepsize implementation. Embedded pairs of methods of Runge-Kutta-Fehlberg type.

Part II

Introduction to the stability of numerical methods. Stiff problems.

Definition of A-stability, AN-stability and BN-stability of a numerical method.

Analysis of A-stability for Runge-Kutta methods: A-stability regions. L-stability.

Analysis of AN-stability and BN-stability for Runge-Kutta methods. Algebraic stability.

The phenomenon of the order reduction: an example. B-convergence.

Introduction to Linear Multistep (LM) methods. Zero-stability and convergence. A-stability, A(α)-stability and stiff-stability. Backward differentiation formulas.

References:
- E. Hairer, S.P. Norsett, G. Wanner: Solving Ordinary Differential Equations I, Nonstiff Problems, Springer-Verlag, Berlin, 1993.
- E. Hairer, G. Wanner: Solving Ordinary Differential Equations II, Stiff Problems, Springer-Verlag, Berlin, 1993.
- J.C. Butcher: Numerical methods for ordinary differential equations. Second edition, John Wiley & Sons, Ltd., Chichester, 2008.
- J.D. Lambert: Numerical methods for ordinary differential systems. John Wiley & Sons, Ltd., Chichester, 1991.
- Lecture notes by the professors.