Numerical methods for Ordinary Differential Equations

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Timetable: 12 hrs (Part I, Prof. Zennaro), Lectures on Thursday, from 3pm to 5pm and Friday, from 10am to 12am. First Lecture on January 17, 2013. 12 hours (Part II, Prof. Vermiglio), First Lecture on March 6, 2013, 11:00. Torre Archimede, Room 2BC/30. See the calendar.

Course requirements: it is advisable to have attended a basic course in Numerical Analysis.

Examination and grading: A unique written exam for both Part I and Part II.

SSD: MAT/08 Numerical Analysis
Aim: We present basic numerical methods for initial value problems in ordinary differential equations and we analyse their convergence and stability properties.

Course contents:
Part I
Existence and uniqueness of the solution and continuous dependence on the data for the initial value problem $y'(x) = f(x, y(x)), y(x_0) = y_0$.
Classical Lipschitz constant and right hand side Lipschitz constant.
General one-step methods; explicit and implicit Runge-Kutta methods.
Definition of local truncation and discretization error for one-step methods and definition of consistency of order p.
Convergence theorem with order p for one-step methods. Order conditions for Runge-Kutta methods. Order barriers for explicit and implicit methods.
Variable stepsize implementation. Embedded pairs of methods of Runge-Kutta-Fehlberg type.

Part II
Introduction to the stability of numerical methods. Stiff problems.
Definition of A-stability, AN-stability and BN-stability of a numerical method.
Analysis of A-stability for Runge-Kutta methods: A-stability regions. L-stability.
Analysis of AN-stability and BN-stability for Runge-Kutta methods. Algebraic stability.
The phenomenon of the order reduction: an example. B-convergence.
Introduction to Linear Multistep (LM) methods. Zero-stability and convergence. A-stability, A(α)-stability and stiff-stability. Backward differentiation formulas.

References:

- E. Hairer, S.P. Norsett, G. Wanner: Solving Ordinary Differential Equations I, Nonstiff Problems, Springer-Verlag, Berlin, 1993.
- E. Hairer, G. Wanner: Solving Ordinary Differential Equations II, Stiff Problems, Springer-Verlag, Berlin, 1993.
- J.C. Butcher: Numerical methods for ordinary differential equations. Second edition, John Wiley & Sons, Ltd., Chichester, 2008.
- J.D. Lambert: Numerical methods for ordinary differential systems. John Wiley & Sons, Ltd., Chichester, 1991.
- Lecture notes by the professors.