Numerical methods for Ordinary Differential Equations

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Timetable: 16 hrs (Part I, Prof. Zennaro), January (see the calendar, Rooms 2AB/45 and 2BC/30) + 12 hours (Part II, Prof. Vermiglio), February (see the calendar, Rooms 2AB/45 and 2BC/30). Torre Archimede.

Course requirements: it is advisable to have attended a basic course in Numerical Analysis.

Examination and grading: A unique written exam for both Part I and Part II.

SSD: MAT/08 Numerical Analysis
Aim: We present basic numerical methods for initial value problems in ordinary differential equations and we analyse their convergence and stability properties.

Course contents:
Part I
Existence and uniqueness of the solution and continuous dependence on the data for the initial value problem $y'(x) = f(x, y(x)), y(x_0) = y_0$.
Classical Lipschitz constant and right hand side Lipschitz constant.
General one-step methods; explicit and implicit Runge-Kutta methods.
Definition of local truncation and discretization error for one-step methods and definition of consistency of order p.
Convergence theorem with order p for one-step methods. Order conditions for Runge-Kutta methods. Order barriers for explicit and implicit methods.
Variable stepsize implementation. Embedded pairs of methods of Runge-Kutta-Fehlberg and Dormand-Prince type.

Part II
Introduction to the stability of numerical methods. Stiff problems.
Definition of A-stability, AN-stability and BN-stability of a numerical method.
Analysis of A-stability for Runge-Kutta methods: A-stability regions. L-stability.
Analysis of AN-stability and BN-stability for Runge-Kutta methods. Algebraic stability.
The phenomenon of the order reduction: an example. B-convergence.
Short analysis of A-stability for linear multistep methods.A(α)-stability and stiff-stability. Backward differentiation formulas.

References:

- E. Hairer, S.P. Norsett, G. Wanner: Solving Ordinary Differential Equations I, Nonstiff Problems, Springer-Verlag, Berlin, 1993
- E. Hairer, G. Wanner: Solving Ordinary Differential Equations II, Stiff Problems, Springer-Verlag, Berlin, 1993
- J.C. Butcher: Numerical methods for ordinary differential equations. Second edition, John Wiley & Sons, Ltd., Chichester, 2008
- J.D. Lambert: Numerical methods for ordinary differential systems. John Wiley & Sons, Ltd., Chichester, 1991
- Lecture notes by the professors